

## Implicit Differentiation

For each problem, use implicit differentiation to find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ .

1)  $2x^3 = 2y^2 + 5$

2)  $3x^2 + 3y^2 = 2$

3)  $5y^2 = 2x^3 - 5y$

4)  $4x^2 = 2y^3 + 4y$

5)  $5x^3 = -3xy + 2$

6)  $1 = 3x + 2x^2y^2$

7)  $3x^2y^2 = 4x^2 - 4xy$

8)  $5x^3 + xy^2 = 5x^3y^3$

9)  $2x^3 = (3xy + 1)^2$

10)  $x^2 = (4x^2y^3 + 1)^2$

11)  $\sin 2x^2y^3 = 3x^3 + 1$

12)  $3x^2 + 3 = \ln 5xy^2$

For each problem, use implicit differentiation to find  $\frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$ .

13)  $4y^2 + 2 = 3x^2$

14)  $5 = 4x^2 + 5y^2$

**Critical thinking question:**

15) Use three strategies to find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ , where  $\frac{3x^2}{4y} = x$ . Strategy 1: Use implicit differentiation directly on the given equation. Strategy 2: Multiply both sides of the given equation by the denominator of the left side, then use implicit differentiation. Strategy 3: Solve for  $y$ , then differentiate. Do your three answers look the same? If not, how can you show that they are all correct answers?

## Implicit Differentiation

For each problem, use implicit differentiation to find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ .

1)  $2x^3 = 2y^2 + 5$

$$\frac{dy}{dx} = \frac{3x^2}{2y}$$

2)  $3x^2 + 3y^2 = 2$

$$\frac{dy}{dx} = -\frac{x}{y}$$

3)  $5y^2 = 2x^3 - 5y$

$$\frac{dy}{dx} = \frac{6x^2}{10y + 5}$$

4)  $4x^2 = 2y^3 + 4y$

$$\frac{dy}{dx} = \frac{4x}{3y^2 + 2}$$

5)  $5x^3 = -3xy + 2$

$$\frac{dy}{dx} = \frac{-y - 5x^2}{x}$$

6)  $1 = 3x + 2x^2y^2$

$$\frac{dy}{dx} = \frac{-3 - 4xy^2}{4x^2y}$$

7)  $3x^2y^2 = 4x^2 - 4xy$

$$\frac{dy}{dx} = \frac{4x - 2y - 3xy^2}{3x^2y + 2x}$$

8)  $5x^3 + xy^2 = 5x^3y^3$

$$\frac{dy}{dx} = \frac{15x^2y^3 - 15x^2 - y^2}{2xy - 15x^3y^2}$$

9)  $2x^3 = (3xy + 1)^2$

$$\frac{dy}{dx} = \frac{-3y^2x - y + x^2}{3x^2y + x}$$

10)  $x^2 = (4x^2y^3 + 1)^2$

$$\frac{dy}{dx} = \frac{-32y^6x^2 - 8y^3 + 1}{48x^3y^5 + 12xy^2}$$

11)  $\sin 2x^2y^3 = 3x^3 + 1$

$$\frac{dy}{dx} = \frac{9x - 4y^3 \cos 2x^2y^3}{6xy^2 \cos 2x^2y^3}$$

12)  $3x^2 + 3 = \ln 5xy^2$

$$\frac{dy}{dx} = \frac{6yx^2 - y}{2x}$$

For each problem, use implicit differentiation to find  $\frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$ .

13)  $4y^2 + 2 = 3x^2$

$$\frac{d^2y}{dx^2} = \frac{12y^2 - 9x^2}{16y^3}$$

14)  $5 = 4x^2 + 5y^2$

$$\frac{d^2y}{dx^2} = \frac{-20y^2 - 16x^2}{25y^3}$$

**Critical thinking question:**

15) Use three strategies to find  $\frac{dy}{dx}$  in terms of  $x$  and  $y$ , where  $\frac{3x^2}{4y} = x$ . Strategy 1: Use implicit differentiation directly on the given equation. Strategy 2: Multiply both sides of the given equation by the denominator of the left side, then use implicit differentiation. Strategy 3: Solve for  $y$ , then differentiate. Do your three answers look the same? If not, how can you show that they are all correct answers?

Strategy 1:  $\frac{dy}{dx} = \frac{6xy - 4y^2}{3x^2}$ , Strategy 2:  $\frac{dy}{dx} = \frac{6x - 4y}{4x}$ , Strategy 3:  $\frac{dy}{dx} = \frac{3}{4}$  To show all

answers are the same, plug  $y = \frac{3x}{4}$  into results for strategies 1 and 2.